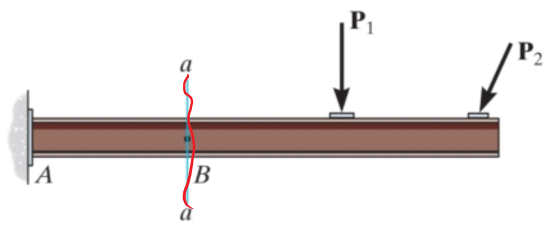
Page 4

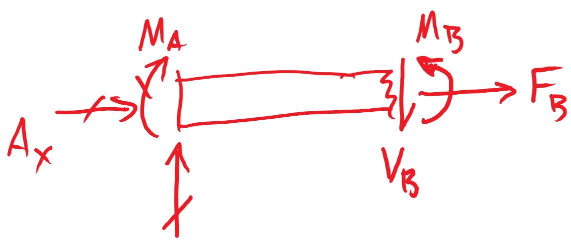
Tuesday, March 7, 2017 7:18 PM

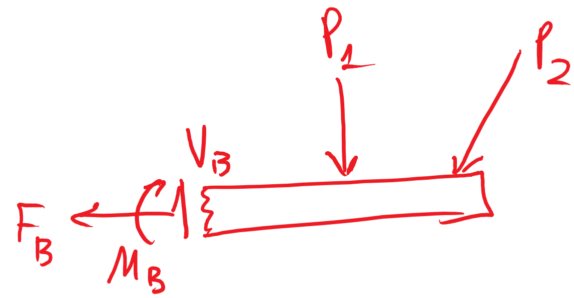
Internal loadings developed in structural members

Structural Design: need to know the loading acting within the member in order to be sure the material can resist this loading

Cutting members at internal points reveal **internal forces and moments.**



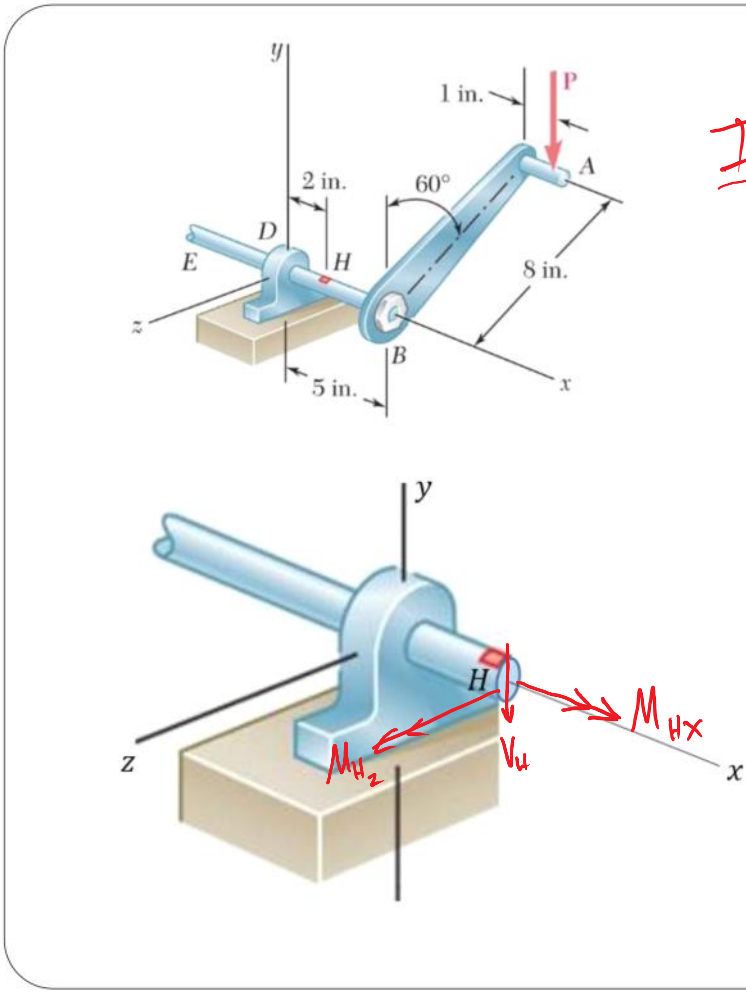




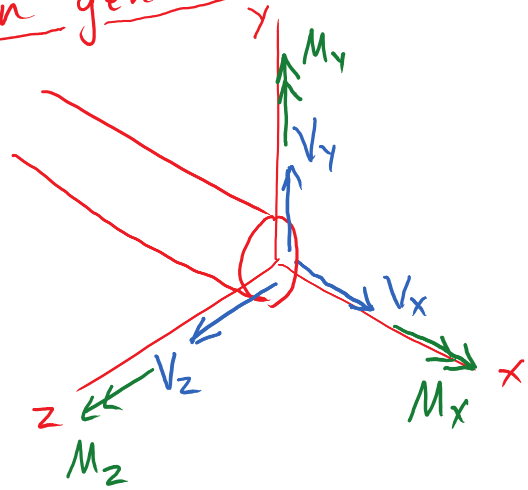
V_B = internal shear force

F_B = internal axial force

M_B = internal bending moment



In general



V_z, V_y are shear forces

V_x is axial force

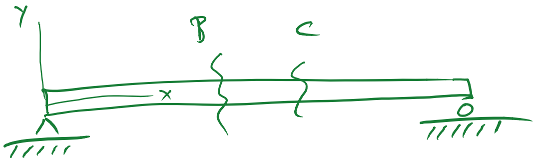
M_z, M_y are bending moments

M_x is a "twisting moment" or a "torque"

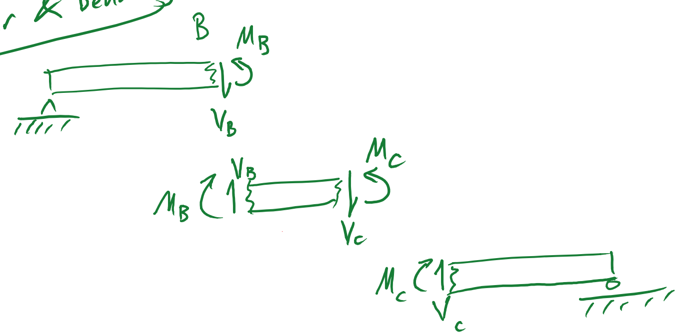
Page 6

Tuesday, March 7, 2017 7:18 PM

Sign conventions:

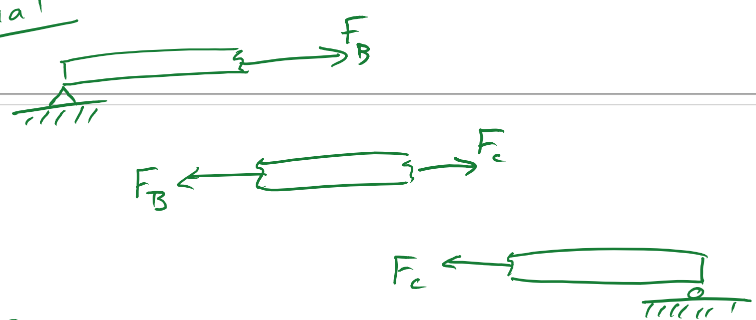


Shear & Bending

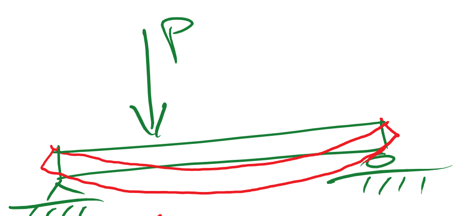


positive sign conventions

Axial



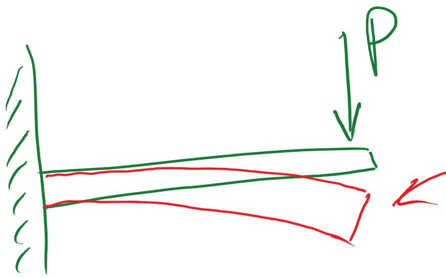
Positive Bending Moment caused the beam to smile!



$M > 0$

⇒ Beam smiles for a positive bending moment

bending moment



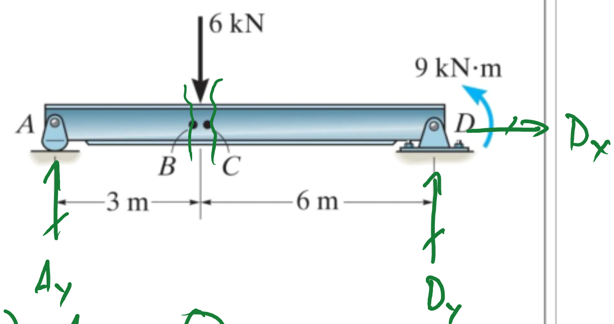
Beam "frowns" for $M < 0$

$\Rightarrow$ Expect M internal to be negative

Procedure for analysis:

1. Find support reactions (free-body diagram of entire structure)
2. Pass an imaginary section through the member
3. Draw a free-body diagram of the segment that has the least number of loads on it
4. Apply the equations of equilibrium

Find the internal forces and moments at B (just to the left of P) and at C (just to the right of P)



$\Sigma F_x = 0 \Rightarrow D_x = 0$

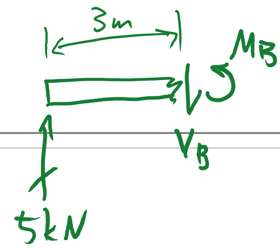
$(\Sigma M)_D = 0 \Rightarrow 9\,kN\cdot m + (6m)(6kN) - (9m)\cdot A_y = 0$

$9\,kN\cdot m + 36\,kN\cdot m = (9m)\cdot A_y \Rightarrow \boxed{A_y = 5\,kN}$

$\Sigma F_y = 0 \Rightarrow A_y + D_y - 6\,kN = 0$

$\Rightarrow \boxed{D_y = 1\,kN}$

Cut @ B



$\Sigma F_y = 0$

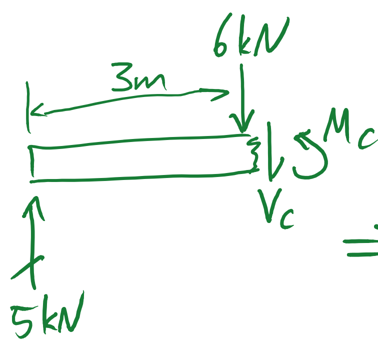
$\Rightarrow 5\,kN - V_B = 0$

$\Rightarrow \boxed{V_B = 5\,kN}$

$(\Sigma M)_B = 0$

$\Rightarrow M_B - (3m)(5\,kN) = 0 \Rightarrow \boxed{M_B = 15\,kN\cdot m}$

Cut @ C



$\Sigma F_y = 0$

$\Rightarrow 5kN - 6kN - V_c = 0$

$\Rightarrow \boxed{V_c = -1\,kN}$

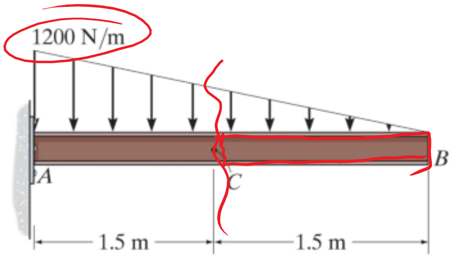
$5\,kN$

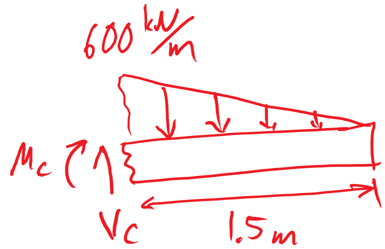
$$\Rightarrow \boxed{V_c = -1\ kN}$$

$$(\Sigma M)_c = 0 \Rightarrow M_c - (3m)(5\,kN) = 0$$

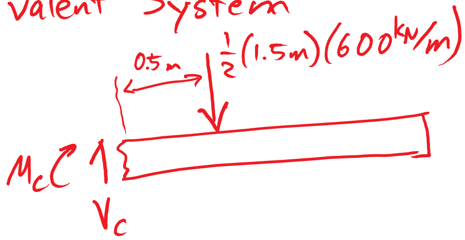
$$\Rightarrow \boxed{M_c = 15\ kN \cdot m}$$

Find the internal forces and moments at C





Equivalent System



$$\Sigma F_y = 0 \Rightarrow V_c - \frac{1}{2}(1.5\,m)(600\,kN/m) = 0$$

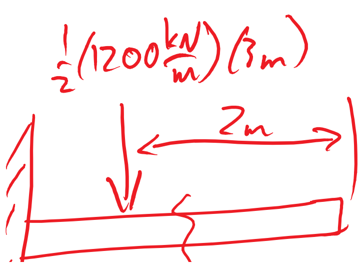
$$\Rightarrow \boxed{V_c = 450\,kN}$$

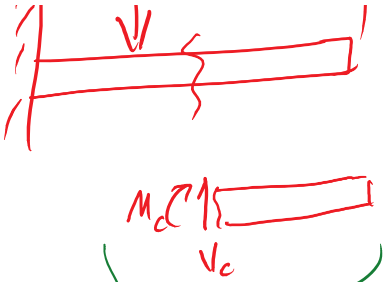
$$(\Sigma M)_c = 0 \Rightarrow -M_c - (0.5\,m)(450\,kN) = 0$$

$$\Rightarrow \boxed{M_c = -225\,kN\cdot m}$$

$M_c < 0 \Rightarrow$ Bending Downward "frowning"

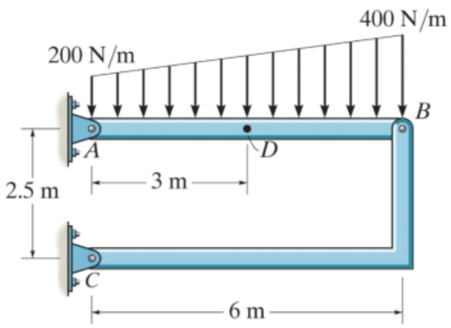
Student Question: Can we "cut" at C after finding the equivalent loading?



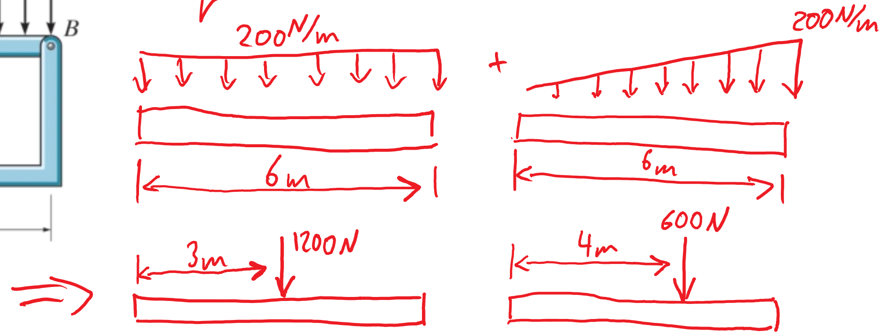


No! If we did, we would get $V_c = 0$, which is <u>not</u> true.

Find the internal forces and moments at D

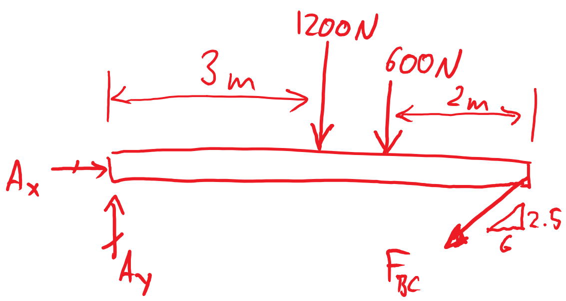


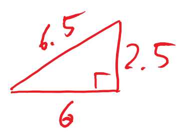
Equivalent load:



Note that CB is a 2-force member

FBD of AB:





Solve ~~for~~ $A_x, A_y, \& F_{BC}$

$$(\Sigma M)_A = 0 \Rightarrow -(3m)(1200N) - (4m)(600N) - (6m)\cdot F_{BC}\cdot\left(\frac{2.5}{6.5}\right) = 0$$

$$-3600\,N\text{-}m - 2400\,N\text{-}m = F_{BC}\left(\frac{15\,m}{6.5}\right)$$

$$\Rightarrow F_{BC} = -\frac{6000\,N}{\left(\frac{15}{6.5}\right)} = -2600N \quad \text{(compression)}$$

$$\Sigma F_y = 0 \Rightarrow A_y - 1200N - 600N - F_{BC}\left(\frac{2.5}{6.5}\right) = 0$$

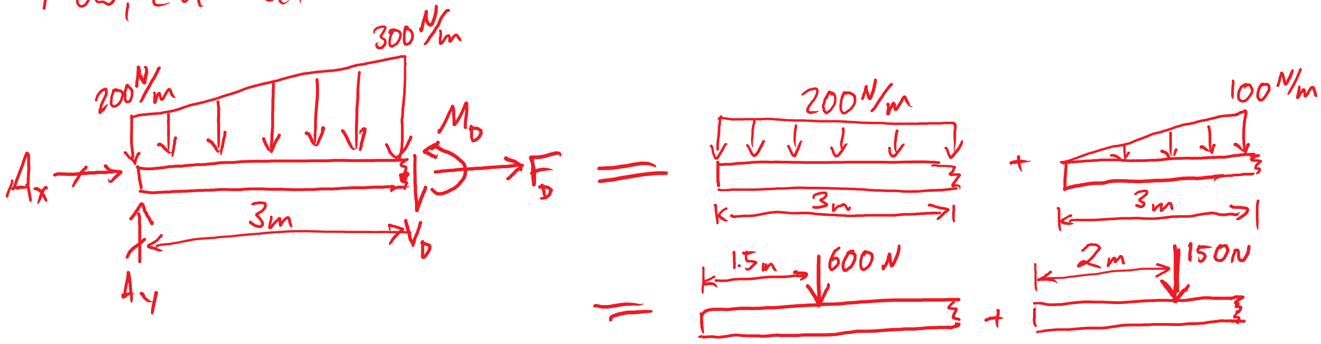
$$A_y - 1800N - (-2600N)\left(\frac{2.5}{6.5}\right) = 0$$

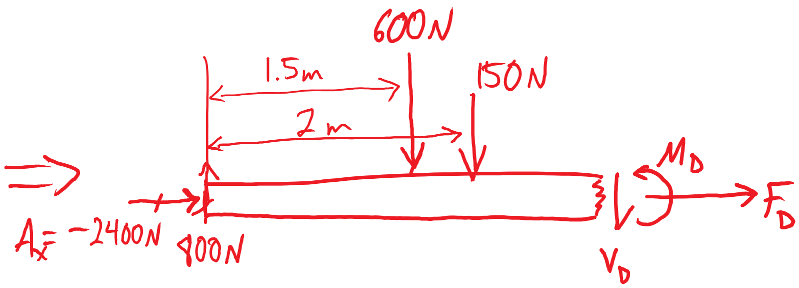
$$\Rightarrow A_y = 800N$$

$$\sum F_x = 0 \Rightarrow A_x - F_{BC} \cdot \frac{6}{6.5} = 0 \Rightarrow A_x = F_{BC} \cdot \left(\frac{6}{6.5}\right)$$

$$A_x = (-2600\,N)\left(\frac{6}{6.5}\right) = -2400\,N$$

Now, cut at D:





$$\sum F_x = 0 \Rightarrow A_x + F_D = 0$$

$$\Rightarrow F_D = -A_x = -(-2400\,N)$$

$$\Rightarrow \boxed{F_D = 2400\,N}$$ Tension.

$$\sum F_y = 0$$

$$\Rightarrow 800N - 600N - 150N - V_D = 0$$

$$\Rightarrow \boxed{V_D = 50\,N}$$

$$\left(\sum M\right)_D = 0$$

$$\Rightarrow M_D + (1m)(150\,N) + (1.5\,m)(600\,N) - (3m)(800\,N) = 0$$

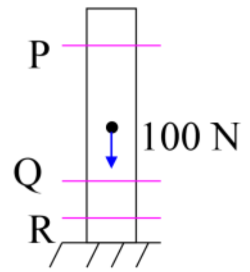
$$M_D = -150\,N\text{-}m - 900\,N\text{-}m + 2400\,N\text{-}m$$

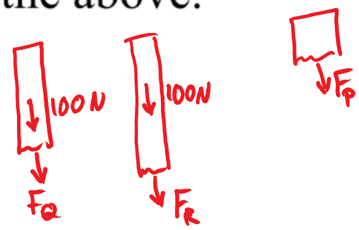
$$\boxed{M_D = 1350\,N\text{-}m}$$

1. A column is loaded with a vertical 100 N force. At which sections are the internal loads the same?

A) P, Q, and R B) P and Q

C) Q and R D) None of the above.





$\Sigma F_y = 0$

$\Rightarrow F_P = 0$

$F_R = F_Q = -100N$

2. Determine the magnitude of the internal loads (normal, shear, and bending moment) at point C.

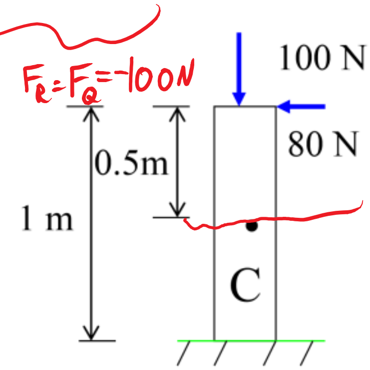
"axial" V M

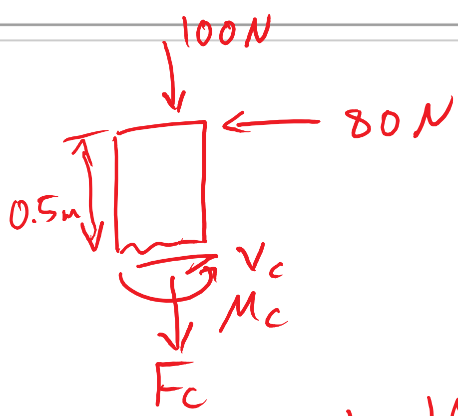
A) (100 N, 80 N, 80 N m)

B) (100 N, 80 N, 40 N m)

C) (80 N, 100 N, 40 N m)

D) (80 N, 100 N, 0 N m)





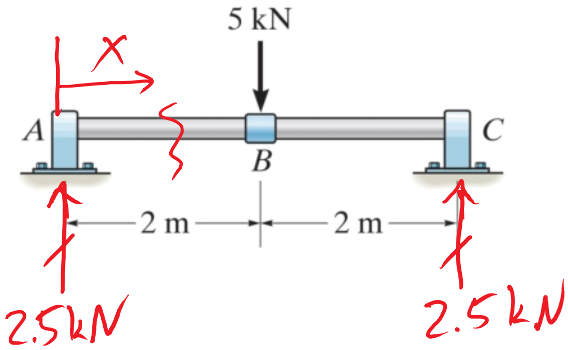
$|F_c| = 100N$

$|V_c| = 80N$

$|M_c| = 40 N\cdot m$

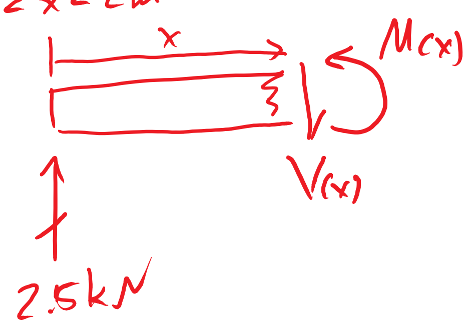
Shear and Moment Equations and Diagrams

The variation in shear force $V(x)$ and bending moment $M(x)$ along a beam is often of interest. The relations for $V(x)$ and $M(x)$ are found from force and moment equilibrium, respectively



Cut at x

$0 < x < 2m$



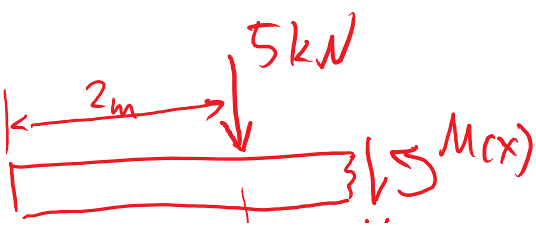
$$\Sigma F_y = 0 \Rightarrow 2.5\,kN - V(x) = 0$$

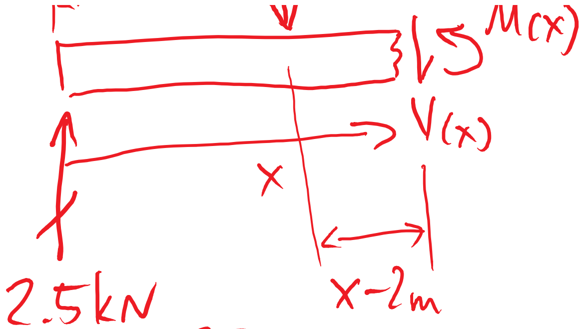
$$\Rightarrow \boxed{V(x) = 2.5\,kN} \quad \text{for } 0 < x < 2m$$

$$(\Sigma M)_x = 0 \Rightarrow M(x) - x \cdot (2.5\,kN) = 0$$

$$\Rightarrow \boxed{M(x) = (2.5\,kN) \cdot x} \quad \text{for } 0 < x < 2m$$

Cut at x, but for $2m < x < 4m$





$$\Sigma F_y = 0$$

$$\Rightarrow 2.5\,kN - 5\,kN - V(x) = 0$$

$$\boxed{V(x) = -2.5\,kN} \quad \text{for } 2m < x < 4m$$

$$(\Sigma M)_x = 0$$

$$\Rightarrow M(x) + (x - 2m)(5\,kN) - x \cdot (2.5\,kN) = 0$$

$$\Rightarrow \boxed{M(x) = (-2.5\,kN) \cdot x + 10\,kN \cdot m} \quad \text{for } 2m < x < 4m$$

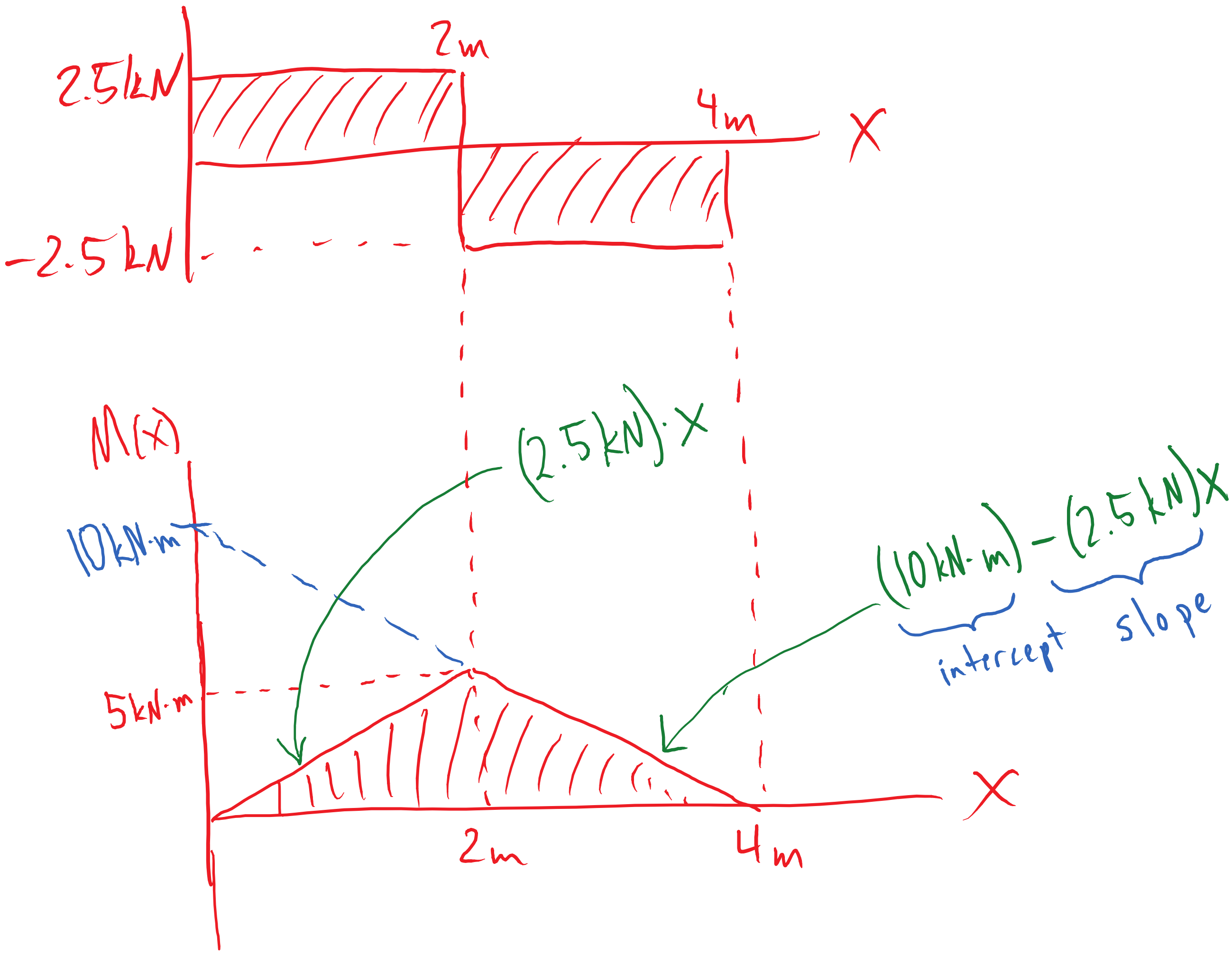
Final answer:

$$V(x) = \begin{cases} 2.5\,kN \,; & 0 < x < 2m \\ -2.5\,kN \,; & 2m < x < 4m \end{cases}$$

Piecewise functions!

$$M(x) = \begin{cases} (2.5\,kN) \cdot x \,; & 0 < x < 2m \\ (10\,kN \cdot m) - (2.5\,kN)x \,; & 2m < x < 4m \end{cases}$$

$V(x)$


2.5kN
2m
4m
X
-2.5kN
M(x)
10kN·m
(2.5kN)·X
$\underbrace{(10kN\cdot m)}_{\text{intercept}} - \underbrace{(2.5kN)}_{\text{slope}}X$
5kN·m
X
2m
4m